MIDTERM 2 (TATARU) - ANSWER KEY

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(1) (a) Eigenvalues: $\lambda = 0, 4, -1$, Characteristic poly: $p(\lambda) = \lambda(\lambda - 4)(\lambda + 1)$

(b)

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 6 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$$

(c)

$$P = \begin{bmatrix} 1 & 9 & 1 \\ -2 & 6 & -1 \\ 1 & 5 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Then you can indeed check that $A = PDP^{-1}$.

(2) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -80 \\ 50 \end{bmatrix}$

(3) Apply Gram-Schmidt to get $\mathcal{B} = \{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ with:

$$\mathbf{w_1} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \mathbf{w_2} = \begin{bmatrix} 16 \\ -4 \\ -4 \\ 9 \end{bmatrix}, \mathbf{w_3} = \begin{bmatrix} -24 \\ -35 \\ 47 \\ 48 \end{bmatrix}$$

(4)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -3 & 2 \end{bmatrix}$$

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(5) (a) **TRUE**

Assume $A\mathbf{v} = \lambda\mathbf{v}$ for some vector $\mathbf{v} \neq 0$. Then:

$$A^2\mathbf{v} = A(A\mathbf{v}) = A(\lambda\mathbf{v}) = \lambda A\mathbf{v} = \lambda\lambda\mathbf{v} = \lambda^2\mathbf{v}$$

However, we also have $A^2 = A$, so $A^2\mathbf{v} = A\mathbf{v} = \lambda\mathbf{v}$.

Hence, we get $\lambda^2\mathbf{v} = \lambda\mathbf{v}$, so $(\lambda^2 - \lambda)\mathbf{v} = \mathbf{0}$.

But since $\mathbf{v} \neq \mathbf{0}$, we get $\lambda^2 - \lambda = 0$, so $\lambda(\lambda - 1) = 0$, hence $\lambda = 0, 1$

(b) Ignore

(c) **FALSE** ($A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable)

(d) **TRUE**

This is slightly tricky!

First of all, since $A^T A = I$, A is **orthogonal**.

But since A is **SQUARE**, this means that A is invertible, and $A^{-1} = A^T$.

However, we know that $AA^{-1} = I$, so $AA^T = I$.

But $A = \left(A^T\right)^T$, so $A^T(A^T)^T = I$.

But this just says that A^T is orthogonal as well! Hence the columns of A^T are orthonormal.

But the columns of A^T are the rows of A, so the rows of A are orthonormal, hence orthogonal!